Nucleation of Superconducting pairing states at mesoscopic scales at zero temperature

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Abstract

We find that spin polarized disordered Fermi liquids are unstable to the nucleation of superconducting pairing states at mesoscopic scales even in magnetic fields substantially higher than the critical one. We study the probability of finding superconducting pairing states at mesoscopic scales in this limit. We find that the distribution function depends only on the film conductance. The typical length scale at which pairing takes place is universal, and decreases when the magnetic field is increased. The number density of these states determines the strength of the random exchange interactions between mesoscopic pairing states.

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The stability of a superconducting state with an order parameter Δ can be characterized in term of the generalized curvature in the following way,

$$\mathcal{O}(\mathbf{r},\mathbf{r}') = \frac{\delta^2 E(\{\Delta(\mathbf{r})\})}{\delta\Delta^*(\mathbf{r})\delta\Delta(\mathbf{r}')}. \tag{1}$$

The curvature determines the stability to the spatial variation of the modulas of the order parameter and the stability to the creation of supercurrents. Here $E(\{\Delta(\mathbf{r})\})$ is the energy of a configuration $\{\Delta(\mathbf{r})\}$. The curvature evaluated at the ground state should be positive defined, i.e. *det* $\mathcal{O}(\mathbf{r},\mathbf{r}') > 0$. For a dirty superconductor where $\sqrt{D/\Delta} \gg l \gg p_F^{-1}$, the curvature at $\Delta(\mathbf{r}) = \Delta$ has mesoscopic fluctuations, like other physical quantities[1]. Here D is the diffusion constant and l is the mean free path, p_F is the Fermi momentum. However, in the absence of an external magnetic field, the mesoscopic fluctuations are small. Thus the curvature is almost positive defined, and the conventional homogenous superconducting state is stable.

When the magnetic field is applied parallel to the disordered thin superconducting film, the suppression of superconductivity is mainly due to Zeeman splitting of electron spin energy levels[2,3]. In the strong spin-orbital scattering limit $\tau_{so}\Delta_0 \ll 1$, close to the critical field H_c^0, the average spin polarization energy E_p is nearly equal to the average condensation energy E_c. Mesoscopic fluctuations of spin polarization energy due to mesoscopic fluctuations of spin sucseptibility become comparable or larger than the energy difference $E_c - E_p$. Therefore, the amplitude of the mesoscopic fluctuations of the curvature evaluated at $\Delta = \Delta(H)$ could be comparable with the average. Here $\Delta(H)$ is the order parameter at given H, Δ_0 is the order parameter for $H = 0$ and $1/\tau_{so}$ is the spin-orbit scattering rate. At $H_c^0 - H/H_c^0 \sim g^{-2}$, where $g = e^2\nu_0 Dd/(2\pi^2\hbar)$ is the dimensionless film conductance, ν_0 the density of states and d the film thickness, the curvature, averaged over the area of size $\xi(H) = \sqrt{D\Delta_0/\Delta^2(H)}$, has a random sign, and the system is unstable with respect to the creation of normal regions or spontaneous creation of supercurrents[4].

One of the consequences of the mechanism discussed above is the *instability of the spin polarized disordered Fermi liquid* well above the critical magnetic field. This is because

though the average curvature of the normal metal state ($\mathcal{O}(\mathbf{r},\mathbf{r}')$ evaluated at $\Delta = 0$)is positive defined, i.e. $E_p > E_c$, its mesoscopic fluctuations have random signs because of the mesoscopic fluctuations of the spin polarization energy. In the regions where the spin polarization energy cost to form superconducting pairing state is much lower than the average energy cost, the fluctuations of the curvature are of large negative value comparable to its positive average such that the normal metal with $\Delta = 0$ becomes unstable. As a result, above the critical field H_c^0, the superconducting pairing correlations are established at mesoscopic scales in the different regions in the normal metal and couple with each other via exchange interactions of random signs.

In this paper, we study the probability to find regions where the superconducting pairing states are formed at mesoscopic scales at $H > H_c^0$. At high magnetic fields in the strong spin-orbit scattering limit, the statistics of these pairing states can be studied with the help of the generalized Landau-Ginsburg equation,

$$\left[\xi_0^2\left(\nabla - i\frac{2e}{c}\mathbf{A}\right)^2 + \frac{H_c^0 - H}{H_c^0}\right]\Delta(\mathbf{r}) + \int \delta\mathcal{O}(\mathbf{r},\mathbf{r}',H)\Delta(\mathbf{r}')d\mathbf{r}' = \frac{\Delta^3(\mathbf{r})}{2\Delta_0^2}, \tag{2}$$

where $\xi_0 = \sqrt{D/\Delta_0}$. The statistical property of the random potential $\delta\mathcal{O}(\mathbf{r},\mathbf{r})$ is determined by its second moment in the Gaussian approximation[4]

$$\langle\delta\mathcal{O}(\mathbf{r}_1,\mathbf{r}_1')\delta\mathcal{O}(\mathbf{r}_2,\mathbf{r}_2')\rangle \propto \frac{1}{g^2}\left[\xi_0^2\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1'-\mathbf{r}_2')\delta(\mathbf{r}_1-\mathbf{r}_1') + \delta(\mathbf{r}_1-\mathbf{r}_2')\delta(\mathbf{r}_1'-\mathbf{r}_2)\frac{\xi_0^4}{|\mathbf{r}_1-\mathbf{r}_1'|^4}\right] \tag{3}$$

where $\langle ...\rangle$ denotes the average over the impurities realizations. The curvature evaluated at the normal metal state where $\Delta(\mathbf{r}) = 0$ has a simple form:

$$\mathcal{O}(\mathbf{r},\mathbf{r}') = -\left[\xi_0^2\left(\nabla - i\frac{2e}{c}\mathbf{A}\right)^2 + \frac{H_c^0 - H}{H_c^0}\right]\delta(\mathbf{r}-\mathbf{r}') + \delta\mathcal{O}(\mathbf{r},\mathbf{r}') \tag{4}$$

Eq.2 is a *nonlinear* equation in terms of $\Delta(\mathbf{r})$, with a *nonlocal* $\delta\mathcal{O}(\mathbf{r},\mathbf{r}')$ potential originating from the oscillations of the wave functions of cooper pairs. Generally speaking, it is qualitatively different from the Schroedinger equation of an electron in the presence of random impurity potentials[5–8]. These complications arise naturally in the study of

the interplay between the mesoscopic effects and the superconductivity and are the generic features of *strongly correlated* mesoscopic systems. In fact, this nonlocal structure of the potential in Eq.2 leads to the superconducting glass state.

However, at $H - H_c^0 \gg H_c^0/g^2$, the optimal configurations which determine the macroscopic properties of the sample turn out to be the superconducting droplets embedded inside the disordered Fermi liquid, with the phases of each droplet coupled via random exchange interaction. Such a configuration can be characterized by three parameters: A). the typical size of the droplet, L_f; B). the typical distance between the droplets, $L_d \gg L_f$; C). the typical value of the order parameter inside each droplet. In the following, we will discuss the statistical of the mesoscopic pairing states in this regime. In this limit, in the leading order of $(L_f/L_d)^2$ the statistical property of the formation of superconducting pairing states at mesoscopic scales is similar to that of the impurity band tails[5–8].

Eq.2 has a nonzero solution only in the regions where the fluctuation of the curvature is of order of average, i.e. $H - H_c^0/H_c^0$ and the corresponding curvature at $\Delta = 0$ is negative. The nonlinear term in Eq.2 determines the amplitude of Δ and its effect will be discussed only when the typical amplitude of Δ is concerned. The most probable configurations are those with superconducting droplets embedded inside the normal metal, i.e. $\Delta(\mathbf{r}) = \sum_\alpha \Delta_\alpha \eta_\alpha(\mathbf{r}), \int d\mathbf{r}\eta_\alpha(\mathbf{r})\eta_\beta(\mathbf{r}) \propto \delta_{\alpha\beta}$. Note $\eta(\mathbf{r})$ introduced in this way is dimensionless. For such a configuration to have lower energy than the normal state, $\int d\mathbf{r}d\mathbf{r}'\Delta(\mathbf{r})\mathcal{O}(\mathbf{r},\mathbf{r}')\Delta(\mathbf{r}') < 0$. The total energy of such a configuration consists of terms corresponding to the coupling between different droplets. The coupling between the droplets decays as distance increases. When the size of the droplets is much smaller than the distance between them, the typical magnitude of the coupling between different droplets is much smaller than that of the coupling within one droplet, provided $L_d \gg L_f$. We are going to neglect such terms in the estimate of the probability of the droplets in the leading order of $o(L_f^2/L_d^2)$. Thus, to have l droplets in the normal metal, l independent inequalities have to be satisfied

$$\Delta_\alpha^2 \left[\int d\mathbf{r}\eta_\alpha(\mathbf{r}) \left(\xi_0^2\nabla^2 + \frac{H - H_c^0}{H_c^0} \right) \eta_\alpha(\mathbf{r}) + \int d\mathbf{r}d\mathbf{r}'\eta_\alpha(\mathbf{r})\delta\mathcal{O}(\mathbf{r},\mathbf{r}')\eta_\alpha(\mathbf{r}') \right] < 0 \tag{5}$$

Furthermore, we can write down the probability to have superconducting pairing states at $H \gg H_c^0$ in term of the sum of probability to have certain number of droplets $\mathcal{P}(\{\eta(x)\}) = \sum_l P_l(\{\eta_\alpha\}|\alpha = 1, ..., l)$. To simplify the notation, we introduce $O_{LG} = \xi_0^2\nabla^2 + H - H_c^0/H_c^0, K_M = \delta\mathcal{O}(\mathbf{r},\mathbf{r}')$. Taking into account $D\eta(\mathbf{r}) = \Pi_\alpha D\eta_\alpha$, we have

$$P_l(\{\eta_\alpha\}|\alpha = 1, ..., l) = \int P(\{K_M\})\Pi_\alpha N^l \int \theta(-L_\alpha + F_\alpha)D\eta_\alpha DK_M \tag{6}$$

where $L_\alpha(\{\eta(\mathbf{r})\}) = \int d\mathbf{r}\eta_\alpha(\mathbf{r})O_{LG}\eta_\alpha(\mathbf{r})$, $F_\alpha(\{\eta(\mathbf{r})\}) = \int d\mathbf{r}d\mathbf{r}'\eta(\mathbf{r})_\alpha K_M(\mathbf{r},\mathbf{r}')\eta_\alpha(\mathbf{r}')$, and N is a normalization constant. We use the following equality to transform the step function into integrals:

$$\theta(-L_\alpha + F_\alpha) = \int_{-\infty}^{0} dg_\alpha \int_{-\infty}^{+\infty} dh_\alpha \exp\left[ih_\alpha\left(L_\alpha - F_\alpha - g_\alpha\right)\right]. \tag{7}$$

Eq. 6 is reduced to $P_l(\{\eta_\alpha\}|\alpha = 1, ..., l) = \Pi_\alpha\rho_\alpha$. In the Gaussian approximation, when the statistics of $\delta\mathcal{O}(\mathbf{r},\mathbf{r}')$ is completely determined by the second moment of the correlation function, ρ_α can be simplified in closed form as

$$\rho_\alpha = N \int \mathrm{erfc}\left[\frac{\int d\mathbf{r}\eta_\alpha(\mathbf{r})O_{LG}\eta_\alpha(\mathbf{r})}{\sqrt{\int d\mathbf{r}_1 d\mathbf{r}_1' \int d\mathbf{r}_2 d\mathbf{r}_2' \mathcal{C}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\eta_\alpha(\mathbf{r}_1)\eta_\alpha(\mathbf{r}_1')\eta_\alpha(\mathbf{r}_2)\eta_\alpha(\mathbf{r}_2')}} \right] D\eta_\alpha \tag{8}$$

where $\mathcal{C}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2') = \langle\delta\mathcal{O}\delta\mathcal{O}\rangle$ as given in Eq.3. $\mathrm{erfc}(a/b) = \int_a \exp(-x^2/2b^2)/\sqrt{2\pi b^2}dx$. One can evaluate the functional integral $D\eta_\alpha(\mathbf{r})$ in the saddle point approximation as long as $H - H_c^0 \gg H_c^0/g^2$. The saddle point equation of Eq.8 can be obtained by minimizing the argument of the error function. The solution of the saddle point equation $\eta_s(\mathbf{r})$ determines the shape of the optimal droplets. To carry out the functional integral of $\eta_\alpha(\mathbf{r})$, one can expand $\eta(\mathbf{r})$ around the saddle point, $\eta(\mathbf{r}) = \eta_s(\mathbf{r}) + \delta\eta(\mathbf{r}), \delta\eta(\mathbf{r}) = \sum_n a_n\eta_n(\mathbf{r})$, where $\eta_n(\mathbf{r})$ are the eigenstates of the operator $\Gamma(\mathbf{r},\mathbf{r}')$ generated via second functional derivative of the argument in the error function with respect to $\eta(\mathbf{r})$ at $\eta(\mathbf{r}) = \eta_s(\mathbf{r})$. Our final result depends slightly on the detail structure of $\Gamma(\mathbf{r},\mathbf{r}')$ and we do not give an explicit form here. Performing the gaussian integral of $\delta\eta(\mathbf{r})$ around the saddle point, taking into account the normalization condition, we obtain,

$$\rho_\alpha = \mathrm{erfc}(\frac{L_s}{\sqrt{2\sigma_s}}) \frac{det' \Gamma(\mathbf{r}, \mathbf{r}')}{det \langle \mathcal{O}(\mathbf{r}, \mathbf{r}') \rangle} \int [da_0] \tag{9}$$

where $L_s/\sqrt{2\sigma_s}$ is the argument of error function in Eq.8 evaluated at $\eta(\mathbf{r}) = \eta_s(\mathbf{r})$. $'$ indicates the exclusion of the zero eigenvalue. The last integral in Eq.9 corresponds to the contribution from the zero eigenvalue state, originating from the translation invariance of the saddle point equation, with $2 - fold$ degeneracy $\eta_{0i}(\mathbf{r}) = L_0 \partial_{0i} \eta_s(\mathbf{r} - \mathbf{r}_0)/\sqrt{\int \eta_s^2 d\mathbf{r}}, i = x, y$[6,7]. Here L_0 is the characteristic length of the droplets determined via the normalization condition $1 = \int d\mathbf{r} L_0^2 (\nabla \eta_s)^2 / 2 \int \eta_s^2 d\mathbf{r}$. Thus,

$$\int [da_0] = \frac{1}{L_0^2} \int_{v_a} dx_\alpha dy_\alpha. \tag{10}$$

The spatial integral is performed only in the region v_α where no other droplets are present. Using the following rescaling

$$\mathbf{r} = y L_f,\ \nabla = \nabla_y L_f^{-1},\ \eta_s(\mathbf{r}) = \eta_s(\frac{y}{L_f}),\ \mathcal{C}(\mathbf{r}, \mathbf{r}'; \mathbf{r}_1, \mathbf{r}_1') = \frac{1}{g^2} \frac{\xi_0^2}{L_f^6} \tilde{\mathcal{C}}(y, y'; y_1, y_1'),\ L_f = \xi_0 (\frac{H_c^0}{H - H_c^0})^{1/2}, \tag{11}$$

we can express L_s, σ_s in term of dimensionless $\eta_s(y)$

$$L_s = B \frac{H - H_c^0}{H_c^0} L_f^2,\ \sigma_s = A^2 \frac{\xi_0^2 L_f^2}{g^2}$$
$$B = \int dy \eta_s(y) (\nabla^2 + 1) \eta_s(y), A^2 = \int dy_1 dy_1' dy_2 dy_2' \tilde{\mathcal{C}}(y_1, y_1'; y_2, y_2') \eta_s(y_1) \eta_s(y_1') \eta_s(y_2') \eta_s(y_2) \tag{12}$$

where B, A^2 are the dimensionless quantities of order of unity depending on the details of $\eta_s(y)$. η_s satisfies the dimensionless saddle point equation

$$(\nabla_y^2 + 1) \eta_s(y) + \int dy_1 dy_1' dy' \tilde{\mathcal{C}}(y, y'; y_1, y_1') \eta_s(y_1) \eta_s(y_1') \eta_s(y') = 0 \tag{13}$$

and at $y = \infty$, $\eta_s(y) = 0$. We also estimate that $det'\ \Gamma(\mathbf{r}, \mathbf{r}')/det \langle \mathcal{O}(\mathbf{r}, \mathbf{r}') \rangle \sim 1$.

Collecting all the results, we have

$$P_l(\{\eta_\alpha\} | \alpha = 1, ..., l) \propto \frac{V^l}{l!} \left\{ \frac{1}{L_f^2} \mathrm{erfc} \left[\frac{Bg}{A} \left(\frac{H_c - H}{H_c} \right)^{1/2} \right] \right\}^l \tag{14}$$

where $V^l/l!$ is from the spatial integral in Eq.10, excluding the overlap between different droplets. We take into account $L_0 \sim L_f$. It is easy to confirm that the average number density of the droplets is

$$\rho = \frac{1}{V}\frac{\sum_l P_l l}{P_l} \propto \frac{1}{L_f^2}\mathrm{erfc}\left[\frac{Bg}{A}\left(\frac{H-H_c^0}{H_c^0}\right)^{1/2}\right]. \tag{15}$$

Let us turn to the problem of the typical amplitude of Δ_α of a droplet. The amplitude of Δ_α is determined by the nonlinear term of Eq.2 and the probability to have a superconducting droplet with $\Delta_\alpha = \Delta$ is

$$\mathcal{P}(\Delta) = N\frac{2\Delta N_\alpha}{\Delta_0^2}\int P(K_M)\delta\left[N_\alpha\left(\frac{\Delta}{\Delta_0}\right)^2 + L_\alpha - F_\alpha\right] dK_M d\eta_\alpha \tag{16}$$

where N_α is given as $N_\alpha = \int \eta_\alpha^4(\mathbf{r})d\mathbf{r}$. L_α, F_α are given after Eq.6. The prefactor in front of the integral is from the Jacobian under the transformation $\delta(x) \to 2Ax\delta(Ax^2)$. Transforming δ-function into a integral and carrying out the gaussian integral of K_M , using the procedure similar to that used in obtaining Eq.8, we can expressed Eq.16 in terms of $\eta_\alpha(\mathbf{r})$. The functional integral can be performed in the saddle point approximation. This saddle point equation turns out to be the same as that of Eq.8 except there is an additional nonlinear term proportional to $N_\alpha(\Delta^2/\Delta_0^2)$. However, as shown below, the typical Δ in the optimal droplet is much smaller than $\Delta_0\sqrt{H-H_c^0/H_c^0}$ in the limit $H - H_c^0/H_c^0 \gg 1/g^2$. Therefore, this nonlinear term is much smaller than the linear term already present in Eq.13 and can be treated as a perturbation as far as the spatial dependence is concerned. As a result, we can use the saddle point solution obtained in Eq.13 to evaluate Eq.16. By expanding the resultant equation in term of the nonlinear term $(\Delta/\Delta_0)^2$ and keeping only the Δ dependent term, we obtain the *conditional* distribution function of Δ of a droplet

$$\mathcal{P}_c(\Delta) = \frac{2Cg^2\Delta}{\Delta_0^2}\exp\left(-Cg^2\frac{\Delta^2}{\Delta_0^2}\right). \tag{17}$$

This shows that $\Delta \sim \Delta_0/g$ is independent of the magnetic field. C is a constant of order of unity. It is much smaller than $\Delta_0\sqrt{H-H_c^0/H_c^0}$, justifying the approximation we made to derive Eq.17.

So far we neglect the coupling between different droplets and treat the droplets as a dilute gas. Though the coupling between droplets does not affect the probability of finding one droplet, it determines the coupling between the different droplets and the global phase

rigidity. The typical distance L_d is order of $L_f \mathrm{erfc}^{-1/2}(Bg/A(H - H_c^0/H_c^0)^{1/2})$ following Eq.15. The typical coupling between two droplets is determined by $\delta\mathcal{O}$ and is given as

$$\left|\nu_0 \Delta_\alpha \Delta_\beta \int d\mathbf{r} d\mathbf{r}' \mathcal{O}(\mathbf{r},\mathbf{r}') \eta_s(\mathbf{r}-\mathbf{r}_\alpha)\eta_s(\mathbf{r}'-\mathbf{r}_\beta)\right| \propto \frac{\Delta_0}{g}\mathrm{erfc}\left[\frac{Bg}{A}\left(\frac{H-H_c^0}{H_c^0}\right)^{1/2}\right]. \qquad (18)$$

To obtain this result, we take into account that the size of the droplet is L_f, typical Δ_α is given by Eq.17, and $|\mathbf{r}_\alpha - \mathbf{r}_\beta| \sim L_d$. The average $\mathcal{O}$, as shown in Eq.3 is proportional to $\delta(\mathbf{r}-\mathbf{r}')$. Thus, the average coupling is proportional to the overlap integral $\int d\mathbf{r}\eta_s(\mathbf{r} - \mathbf{r}_\alpha)\eta_s(\mathbf{r}-\mathbf{r}_\beta) \propto \exp(-L_d/L_f)$, which is small in the limit $L_d \gg L_f$. This indicates that the distribution function of the coupling between different pairing states is symmetric with respect to zero.

The existence of random Josephson coupling in the presence of a parallel magnetic field is a consequence of the Pauli spin polarization. This phenomena exists even without spin orbit scattering. Consider for example a granular superconductor, with superconducting grains coupled with each other via Josephson coupling. The sign of the Josephson coupling is determined by the total phase of the time reversal pairs. In the pure limit, though the sign of the wave function of each electron oscillates with a period of Fermi wave length, the total phase of $(\mathbf{p}, -\mathbf{p})$ pair is zero because of the exact cancellations of the phases of each electron inside the pair. Therefore there is no sign oscillation for Josephson couplings. In the dirty case $\mathbf{p}$ is not a good quantum number. However the sign of the coupling doesn't oscillate as a function of spatial coordinate because of the time reversal symmetry. As a result, even when the distance between the grains is much larger than the mean free path, the sign of the coupling is positive definite[9]. This is in contrast to the ferromagnetic ordering nuclear spins due to RKKY exchange interaction. In the pure limit, RKKY coupling exhibits Friedel oscillations with the period of Fermi wave length. In the presence of impurity scatterings, the phase of Friedel oscillations of electron wave functions becomes random. As a result, the nuclear spin system exhibits spin-glass type ordering instead of ferromagnetic ordering when the impurities are present.

In the presence of a parallel magnetic field, the electrons inside the normal metal become

polarized. In this case, the electron with spin up has a different energy as the electron with spin down at the Fermi surface because of the Pauli spin polarization. As a result, the phase of the electron spin up does not cancel with that of spin down in the presence of Zeeman splitting, and the total phase is equal to $\int \mathbf{p}_{up} \cdot d\mathbf{r} + \int \mathbf{p}_{down} \cdot d\mathbf{r} = \int dr \mu_B H / v_F$. Here μ_B is Bohr magnetor and v_F is the Fermi velocity. This leads to the sign oscillations of the Josephson coupling. We assume the electrons in the metal are fully polarized but neglect the spin polarization effect inside the grain because the Zeeman splitting energy scale is much smaller than the energy gap inside the grain. More specifically, Josephson coupling between these two superconducting grains is proportional to the cooper pair correlation function[10,11] $T\sum_n \{\sigma_y^{\alpha\beta} G_{\epsilon_n}^{\beta\gamma}(\mathbf{r}_1, \mathbf{r}_2) \sigma_y^{\gamma\delta} G_{-\epsilon_n}^{\delta\alpha}(\mathbf{r}_1, \mathbf{r}_2)\}$, where σ_y is the y component Pauli matrix, $G_{\epsilon_n}^{\beta\gamma}$ is the Green function in Mastubara representation with spin index β, γ. In the clean limit when $l \gg \mu_B H / v_F$ at $T = 0$, the cooper pair correlation function is

$$\int d\epsilon d\theta \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{\cos\left[\mathbf{Q} \cdot (\mathbf{r}_1 - \mathbf{r}_2)\right]}{2\epsilon + 2\mu_B H + \mathbf{v}_F \cdot \mathbf{Q}} \tanh(\frac{\epsilon}{2kT}) \propto \cos\left(|\mathbf{r}_1 - \mathbf{r}_2| \frac{2\mu_B H}{v_F} - \frac{\pi}{4}\right) \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|^2} \quad (19)$$

for $2D$ case at large distances. θ is the angle between Fermi velocity and $\mathbf{Q}$. The sign of the coupling oscillates with a period $v_F / \mu_B H$, which is much longer than the Fermi wave length, with which the sign of RKKY interaction oscillates.

In disordered metals, for electron pairs to travel between grains separated with a distance L, they must typically move along paths of length of order of L^2/l. When $L^2/l \ll v_F/\mu_B H$, the sign is unpredictable. Indeed in the dirty limit when $L \gg \sqrt{D/\mu_B H} \gg l$, the Josephson coupling averaged over the impurity configuration is exponentially small $\exp(-\sqrt{2}|\mathbf{r}_1 - \mathbf{r}_2|/\sqrt{D/\mu_B H})$ while the typical amplitude of the coupling decays in the same way as in the pure limit,i.e. $\langle |E_J| \rangle \propto |\mathbf{r}_1 - \mathbf{r}_2|^{-2}$. Therefore when the magnetic field increases, only the position of the maximun of the distribution function moves towards zero while the width of the distribution function barely changes. This results in superconducting glass states. Note that in principal the charging effect inside the grain will also lead to superconducting glass phase as suggested in a recent experiment[12]. However in the metallic limit when the tunneling conductance between the grain and the normal metal is

much larger than $e^2/\hbar$ such an effect is negligible.

It is worth pointing out the mesoscopic pairing states we discussed in this paper are not originating from the inhomogeneity of the impurity concentrations. In fact, the range of the impurity potential is of atomic scale due to the perfect screening in the metal and the fluctuation of the impurity concentration within the area of coherence length is inversely proportional to $\sqrt{n_{im} d\xi_0^2}$, and becomes negligibly small in the disordered metal. Here n_{im} is the impurity concentration. Most importantly, the long range coupling mentioned above, which leads to the superconducting glass state, is not related to fluctuations of any local quantities. We are very grateful to B. Altshuler, L. I. Glazman, D. Huse, I. E. Smolyarenko, B. Spivak for useful discussions. F .Zhou is supported by Princeton University. C. Biagini is supported under PRA97-QTMD in Italy. We also like to thank NEC Research Institute for its hospitality.

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